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DERIVATION OF A SIMPLIFIED MATHEMATICAL MODEL OF A DMU'S POWERTRAIN

ODVODENIE ZJEDODUŠENÉHO MATEMATICKÉHO MODELU HNACEJ SÚSTAVY DM JEDNOTKY

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1 INTRODUCTION

Recently, railway transport means can be divided to two main groups depending on the source of the energy. Although dependent traction rail vehicles are more and more popular and many European countries endeavour to build new and new track form them, they cannot operate in every corner of the country [1, 2]. There are regions of countries, where the building of track with electrical network is it is not worth it, it is too difficult or there are other reasons not to not to expand the electrical network for electric rail vehicles [3, 4]. Independent traction rail vehicles are nowadays the most known as train units (***fig. 1***).

Fig. 1 *An example of a DMU produced in the Slovak Republic*

Obr. 1 *Príklad DM jednotky vyrábanej v Slovenskej republike*

The presented research is a part of the complex project research focused on the investigation phenomena in the transmission system of a railway unit under a change of power conditions. The main objective is to derive a computational model of a transmission system such a railway train unit. This contribution includes the first step. Although the diesel-mechanical transmission system with a flexible clutch or with a friction lamellar clutch is recently one of the oldest transmission systems, the derivation of its transmission system operation requires to understand the fundamentals of the used methods. Therefore, a presentation of the achieved findings offers the scholars and researchers to see the considered procedure for other further activities, which will lead to creation of the entire railway train unit with the modern and environmental friendly power transmission.

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2 DERIVATION OF MATHEMATICAL MODELS

The main purpose of this research is to create a mathematical model of the diesel-mechanical transmission of a railway vehicle. The diesel engine, which powers an unit, can be placed above a frame or it is located under a floor. A control station can be at both ends, on one end or in a separate wagon. Usually, diesel mechanical multiple units (DMUs) are classified by the way, how the motive power is transmitted to their wheels. In case of a diesel mechanical multiple unit (DMMU), mechanical energy of the engine is transmitted to the wheels by means of a gearbox and driveshaft. It is like a road car. In principle, the gear ratio can be shifted manually by a driver. However, this technical solution is in these days considered out-of-date. In the most applications, gears are changed by means of an automatic gearing system [5-7].

The mathematical model of the transmission system comprises equations of motion. From the mathematical point of view, they are differential equations, and they are derived by means of a suitable method. In our case, the method of the Lagrange's equations of a second kind seems to be a suitable method. The general form of the Lagrange's equations of the second kind are as following:

$$\frac{d}{dt}\frac{\partial E_K}{\partial \dot{q}_j} - \frac{\partial E_K}{\partial q_j} + \frac{\partial E_D}{\partial \dot{q}_j} + \frac{\partial E_P}{\partial q_j} = Q_j \ , \ j = 1, 2, ..., n \ , \qquad (1)$$

where E_k is kinetic energy, E_d is dissipative energy, E_p is potential energy, q_j is a generalized coordinate, $\dot{q}_j$ is a generalized velocity and Q_j represents the external load of the system.

A letter n expresses the number of degrees of freedom. As it can be seen from eq. 1, this method needs to know number of degrees of freedom. They directly depend on chosen the number of generalized coordinates.

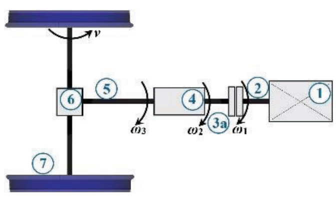

Fig. 2 *A simplified scheme of a mechanical transmission system with a flexible clutch*

Obr. 2 *Zjednodušená schéma hnacej sústavy s pružnou spojkou*

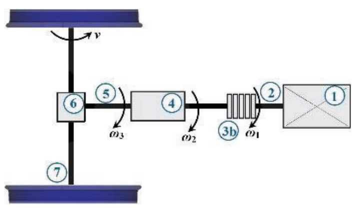

Fig. 3 *A simplified scheme of a mechanical transmission system with a friction lamellar clutch*

Obr. 3 *Zjednodušená schéma hnacej sústavy s trecou lamelovou spojkou*

A simplified scheme of a mechanical system of the diesel-mechanical transmission with a flexible clutch can be seen in ***fig. 2***. It comprises of relatively separate components as following: an engine (1), a crankshaft (2), a flexible clutch (3a), a gearbox (4), an output shaft (5), a distribution gearbox (6) and a driven wheelset (7). All components are considered rigid except of the clutch. It means, that this mechanical system is described by means of two generalized coordinates. The relative motion occurs just in the flexible clutch [8]. The generalized coordinates are angles of rotation of the clutch input shaft φ_1 and the clutch output shaft φ_2. The angle of rotation φ_3 is the coordinate, which relates with the gearbox output shaft. Hence, the mechanical system has two degrees of freedom (2 DOF) and these independent angular motions are described by the coordinates φ_1 and φ_2. When one considers a simplified scheme of a powertrain, the needed energies for the derivation of the equations of motion are as following:

- Kinetic energy:

$$E_K = \frac{1}{2}\cdot\left(I_1\cdot\dot{\varphi}_1^2 + I_2\cdot\dot{\varphi}_2^2 + I_3\cdot\dot{\varphi}_3^2 + m_t\cdot v^2\right) = \frac{1}{2}\cdot\left[I_1\cdot\dot{\varphi}_1^2 + \left(I_2 + \frac{I_3}{i_t^2} + \frac{m_t\cdot R^2}{i_t^2}\right)\cdot\dot{\varphi}_2^2\right] \tag{2}$$

where I_1, I_2 and I_3 and moments of inertia of the components of the system, which moves at the rotational velocity $\dot{\varphi}_1$, $\dot{\varphi}_2$, and $\dot{\varphi}_3$, respectively, m_t is the total weight of the vehicle, R is the wheel diameter and i_t is the total gear ratio of the transmission system, at which $i_t = i_g\cdot i_{dg}$, where i_g is the gear ratio of the active grade and i_{dg} is the gear ratio of the distribution gearbox. It should be noted, that it is considered the same peripheral speed of the wheel as the vehicle in a centre of gravity and no wheel slip. Further, equation 2 shows an expression of the other coordinates, or kinematic quantities, i.e. φ_3 and v by means of the defined generalized coordinates φ_1 and φ_2. Thus, the following relations are valid:

$$\frac{\dot{\varphi}_2}{\dot{\varphi}_3} = i_t \;\Rightarrow\; \dot{\varphi}_3 = \frac{\dot{\varphi}_2}{i_t}; \quad v = R\cdot\dot{\varphi}_3 \;\Rightarrow\; v = R\cdot\frac{\dot{\varphi}_3}{i_t}. \tag{3}$$

- Dissipative energy:

$$E_d = \frac{1}{2}\cdot b_1\cdot\dot{\varphi}_1^2 + \frac{1}{2}\cdot b_2\cdot\dot{\varphi}_2^2 \tag{4}$$

where coefficients b_1 and b_2 represent viscous losses in the system,

- Potential energy:

$$E_p = \frac{1}{2}\cdot k\cdot\left(\varphi_2 - \varphi_1\right)^2 \tag{5}$$

where k is the torsional stiffness of the flexible clutch and it is supposed, that $\varphi_2 > \varphi_1$. The potential energy is given by a simple formulation (eq. 5), therefore it is considered a vehicle running on a straight track without inclinations. The right side of the eq. 1 includes the external loads of the system. In our case, these loads are given by the moments acting to the individual components. The crankshaft is loaded by the driving torque of the engine M_k and the output shaft from a gearbox is loaded by the moment of resistances M_r. Although the moment M_r acts on the component (6), it should be also converted to the shaft (3a), i.e. to the component rotating by the angular velocity $\dot{\varphi}_2$. After calculation of the derivation of the individual energies and considering the described assumptions, the resulting mathematical model of the diesel-mechanical power transmission of a railway vehicle with the flexible clutch is:

$$\begin{gathered} I_1\cdot\ddot{\varphi}_1 + b_1\cdot\dot{\varphi}_1 + k\cdot\varphi_1 - k\cdot\varphi_2 = M_k\ , \\ \left(I_2 + \frac{I_3}{i_t^2} + \frac{m_T\cdot R^2}{i_t^2}\right)\cdot\ddot{\varphi}_2 + b_2\cdot\dot{\varphi}_2 - k\cdot\varphi_1 + k\cdot\varphi_2 = -M_r\ , \end{gathered} \tag{6}$$

or the matrix form:

$$\begin{bmatrix} I_1 & 0 \\ 0 & \left(I_2 + \frac{I_3}{i_t^2} + \frac{m_t\cdot R^2}{i_t^2}\right) \end{bmatrix}\cdot\begin{bmatrix}\ddot{\varphi}_1 \\ \ddot{\varphi}_2\end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2\end{bmatrix}\cdot\begin{bmatrix}\dot{\varphi}_1 \\ \dot{\varphi}_2\end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k\end{bmatrix}\cdot\begin{bmatrix}\varphi_1 \\ \varphi_2\end{bmatrix} = \begin{bmatrix} M_k \\ -M_r\end{bmatrix} \tag{7}$$

A transmission system with a friction lamellar clutch is shown in ***Fig. 3***. In this case, a friction moment is considered in the system as following:

$$M_c = S\cdot p\cdot f\cdot n\cdot R_c \tag{8}$$

where $\boldsymbol{S}$ is the clutch piston area, $\boldsymbol{p}$ is the instantaneous oil pressure, $\boldsymbol{f}$ is the friction coefficient and $\boldsymbol{R_c}$ is the mean radius of the clutch slats. The determined energies of the system, i.e. kinetic energy $\boldsymbol{E_k}$ (eq. 2), dissipative energy $\boldsymbol{E_d}$ (eq. 4) and potential energy $\boldsymbol{E_p}$ (eq. 5) for the system with the flexible clutch are in principle the same for the system with friction lamellar clutch. When the friction moment $\boldsymbol{M_c}$ is considered (eq. 8), the system of equations of motion is as following:

$$I_1 \cdot \ddot{\varphi}_1 + b_1 \cdot \dot{\varphi}_1 + k \cdot \varphi_1 - k \cdot \varphi_2 = M_k - M_c \ ,$$
$$\left(I_2 + \frac{I_3}{i_t^2} + \frac{m_T \cdot R^2}{i_t^2} \right) \cdot \ddot{\varphi}_2 + b_2 \cdot \dot{\varphi}_2 - k \cdot \varphi_1 + k \cdot \varphi_2 = M_c - M_r \ , \tag{9}$$

or the matrix form:

$$\begin{bmatrix} I_1 & 0 \\ 0 & \left(I_2 + \frac{I_3}{i^2} + \frac{m_t \cdot R^2}{i^2} \right) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \cdot \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} M_k - M_c \\ M_c - M_r \end{bmatrix} \tag{10}$$

The calculation of the derived equations of motion of the system with the flexible clutch (eq. 7) and the equations of motion of the system with the friction lamellar clutch is performed by means of the technical programming language Matlab [9, 10]. We calculated the equations of motion in the time domain for the prescribed initial conditions. For both simplified models, we considered the following initial conditions:

$$t = 0: \qquad \varphi_1 = \varphi_2 = 0; \quad \dot{\varphi}_1 = \dot{\varphi}_2 = 0 \tag{11}$$

3 RESULTS AND DISCUSSION

As it was mentioned above, the created mathematical models of the transmission system with the flexible clutch (eq. 7) and the transmission system with the friction lamellar clutch (eq. 10) have been solved in the time domain in the Matlab software.

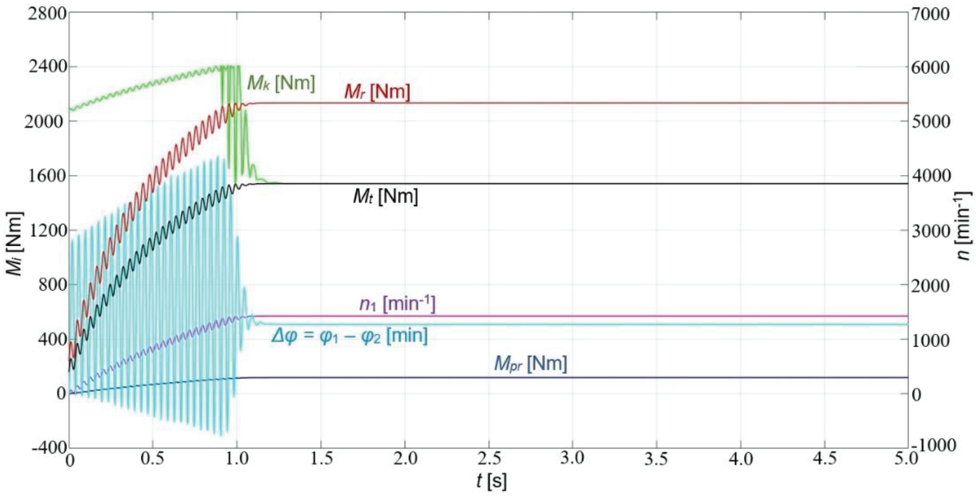

Fig. 4. *Waveforms of the output quantities of system with the flexible clutch*

Obr. 4 *Priebeh výstupných veličín systému s pružnou spojkou*

The achieved results are shown in the form of graphs, which depict the waveform of selected output quantities. ***Fig. 4*** shows a waveform of the output quantities of the diesel-mechanical transmission with the flexible clutch. This graph includes five curves, which are marked as following: $\boldsymbol{M_k}$ – the engine torque [Nm], $\boldsymbol{M_r}$ – the resistance moment [Nm], $\boldsymbol{M_t}$ – the total moment of the load [Nm], $\boldsymbol{M_{pr}}$ – the moment of passive resistances [Nm], $\boldsymbol{n_1}$ – rotations of the crankshaft 2 [min^{-1}] (***Fig. 2***) and $\boldsymbol{\Delta\varphi = \varphi_1 - \varphi_2}$ – the relative deflection of both part of the flexible clutch [min].

The time interval of 5 seconds has been chosen for illustration of the outputs. The moment $\boldsymbol{M_r}$ represents the running resistance of the vehicle. The moment of passive resistances includes the losses in the bearings, gears, and other components (friction, rolling etc.). As it can be seen, the moment of the engine raises from the initial value of app. 1500 Nm during the time interval of app. 0.8 s. Together with it, the moment of resistance $\boldsymbol{M_r}$ also raises. At the certain value of these moments, the moments equilibrium is achieved. It is app. after 1.15 s. After this time, the equilibrium of the mechanical system of the transmission is achieved and the values of moments and rotations of the crankshaft are constant. This steady-state would be disturbed in case of a change of the load, e.g. change of running resistance, i.e. a change of the moment $\boldsymbol{M_r}$ or $\boldsymbol{M_t}$. In interesting output is the relative deflection of the flexible clutch parts (output $\boldsymbol{\Delta\varphi}$). As it can be seen, during the unbalanced state, which corresponds with the start of the mechanical system of the transmission, this angle changes to the +/- values. It is caused by the clutch flexibility it is observed during the time interval of 1.15 s. When the stead-state is achieved (after 1.15 s), the relative deflection of the clutch parts disappears and both parts of the flexible clutch rotate with the same rotations (the time interval 1.15 s to 5 s).

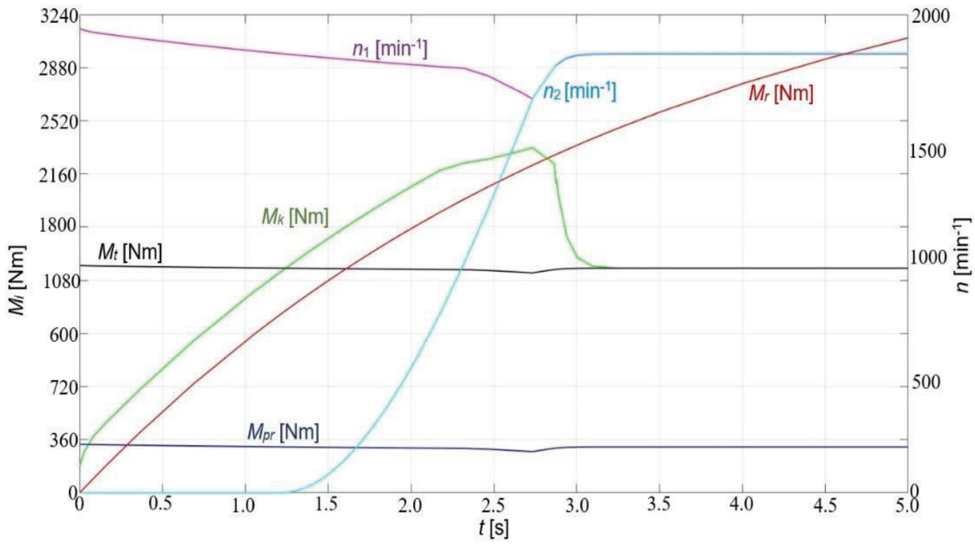

Fig. 5. *Waveforms of the output quantities of system with the friction lamellar clutch*

Obr. 5 *Priebeh výstupných veličín systému s trecou lamelovou spojkou*

Waveforms of the output quantities of the diesel-mechanical transmission system of a rail vehicle are shown in ***fig. 5***. In this case, we have observed six output quantities. There are namely following outputs: $\boldsymbol{M_k}$ – the engine torque [Nm], $\boldsymbol{M_r}$ – the resistance moment [Nm], $\boldsymbol{M_t}$ – the total moment of the load [Nm], $\boldsymbol{M_{pr}}$ – the moment of passive resistances [Nm], $\boldsymbol{n_1}$ – rotations of the crankshaft 2 [min^{-1}] (***Fig. 3***) and $\boldsymbol{n_2}$ – rotations of the lamellar clutch output shaft [min^{-1}] (***Fig. 3***).

The different behaviour the diesel-transmission system with the friction lamellar clutch is obvious at the first glance. The graph shows these outputs again in the time interval of 5 s. It can be seen that the torque onset of the crankshaft M_k is not so sharp in comparison with the flexible clutch. Similarly, the waveforms of the other moments are smoother. The rotations of the crankshaft n_1 achieve the equilibrium with the rotation n_2 of the output shaft of the clutch after 2.75 seconds. It can be also seen that the torque and the moment of the load achieve the equilibrium after app. 3.15 seconds. The smoother onset of the moments and rotations of the shafts can be explained by the working principle of the friction lamellar clutch. The clutch transmits the torque through its slats, at which, certain slip appears. This slip makes smoother the waveforms of the moments. The pressure between slats achieves the maximal value and then, the moments (torque and the resistance moments) are equal as well as the rotations of the shafts n_1 and n_2. The friction lamellar clutch shows smoother operation of the transmission system, which contributes to the higher ride comfort for passengers, better control of the moments in the system and higher protection against overloading.

The future research will be focused on creation of mathematical models of other types of DMU, namely on DHMU and DEMU. These considered models of the transmission system seem to be more complicated. The hydraulic system working in the transmission system needs to be described by more complicated model including change of pressure in the hydrodynamic converter. The operational principle of the DEMU, which are the most widely used DMU, will be even more complicated. It relates with the fact, that the combustion engine powers a generator and then, the electricity power electromotors in driven bogies. The creation of the multibody model of the transmission system of a railway vehicle is a perspective solution of this problem [11, 12]. The final step of the research, the results of the mathematical models form the Matlab and from the multibody models will be compared with the perspective of achieving the satisfying results.

The performed research includes the fundamental procedures and principles, which can be applied for more difficult problems focused on the investigation of dynamics of transmission systems. These systems are used for driven of railway vehicles. Despite the flexible clutch and lamellar clutch are used in transmission system of older railway vehicles and for railway vehicles with lower power, the presented mathematical models is important for understating the background of the principle and application for other steps of the more complex dynamical systems of railway vehicles.

4 CONCLUSION

The railway vehicles with the independent source of power are still used in some countries. There are locomotives or railway train units. These railway vehicles use several types of transmission systems. They differ to each other in their complexity, purpose of the applications, requirements for transmitted power, torque and others. One of the simplest transmission system a railway vehicle with the independent powertrain is the diesel-mechanical transmission system. It includes an engine, a clutch, a gearbox and other needed components. In case of investigation of the operational conditions of these railway vehicles, a mathematical model is needed. This contribution was focused on the derivation of mathematical model of a diesel-mechanical transmission system of a railway vehicle. The mathematical model was derived by means of the Lagrange's equations of the second kind. There were set-up two mathematical models, namely for the transmission system with the flexible clutch and with the friction lamellar clutch. Based on the mathematical models, the simulations of the start of these transmission systems were performed. The results of the simulations were presented in the form of graphical outputs. The achieved results shown differences of the waveform of the individual outputs quantities. The presented procedure can be applied for a derivation of mathematical models of the more complex transmission system of a railway vehicles.

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Summary

Railway transport belongs to the most common kinds of transport. Railways provide transportation of large amount of goods as well as huge number of passengers around the world. The reliable movement of railway vehicles is ensured by means of the power transmission system. It includes various components. Several types of components are distinguished. It depends on the way, how the power is transmitted from the power source to the driven wheels of a railway vehicle. In principle, there are known mechanical, hydraulic and electric ways for the power transmission. The simplest type is the diesel-mechanical power transmission. Although, the diesel-mechanical power transmission represents the *oldest type of the power transmission in case of railway vehicles, it is still possible to find it in some type of railway vehicles called as the DMUs. The presented article is focused on the derivation of a mathematical model of the diesel-mechanical power transmission of a railway vehicle. It is derived by means of the Lagrange's equations of the second kind. The derived mathematical model allows to understand the fundamentals of the derivation of the mathematical model of the diesel-mechanical power transmission of a railway vehicle, an application of the numerical mathematical methods for the solving the equations of motion and for evaluation of the wanted outputs quantities, such as waveforms of torques, revolutions of shafts and others.*

Resumé

Železničná doprava patrí k najbežnejším druhom dopravy. Železnice zabezpečujú prepravu veľkého množstva tovaru, ako aj veľkého počtu množstva cestujúcich po celom svete. Spoľahlivý pohyb železničných vozidiel je zabezpečený prostredníctvom systému prenosu energie. Ten zahŕňa rôzne komponenty. Rozlišuje sa niekoľko typov komponentov. Závisí to od spôsobu prenosu výkonu zo zdroja na poháňané kolesá železničného vozidla. V zásade sú známe mechanické, hydraulické a elektrické spôsoby prenosu sily. Najjednoduchším typom je diesel-mechanický prenos výkonu. Aj keď diesel-mechanický prenos sily predstavuje najstarší typ prenosu sily v prípade železničných vozidiel, stále je ho možné nájsť v niektorých typoch železničných vozidiel nazývaných DMU. Predkladaný článok je zameraný na odvodenie matematického modelu diesel-mechanického prenosu výkonu železničného vozidla. Odvodzuje sa pomocou Lagrangeových rovníc druhého druhu. Odvodený matematický model umožňuje pochopiť základy odvodenia matematického modelu diesel-mechanického prenosu výkonu železničného vozidla, aplikáciu numerických matematických metód na riešenie pohybových rovníc a na vyhodnotenie požadovaných výstupných veličín. , ako sú priebehy krútiacich momentov, otáčky hriadeľov a ďalšie.