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DERIVATION OF A SIMPLIFIED MATHEMATICAL MODEL OF A RAIL VEHICLE POWERTRAIN WITH A HYDRODYNAMIC CONVERTER

ODVODENIE ZJEDNODUŠENÉHO MATEMATICKÉHO MODELU HNACEJ SÚSTAVY KOĽAJOVÉHO VOZIDLA S HYDRODYNAMICKÝM MENIČOM

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1 INTRODUCTION

In these days, railway transport is one of the most often used kind of transport of people and goods for shorter as well as longer distances. This is not only in many European countries, but also in other part of the world. Day by day people commute to work and to home by railway transport means, such as subway systems, light rail transits as well as by other kinds of railway transport means. The railway transport has several advantages, which make it effective, reliable, and sustainable in comparison with other kinds of transport [1]. On one hand, there are rail vehicles, which are powered from power line. On the other hand, there are rail vehicles, which has own source of power. The rail vehicles of the first group are called as dependent traction vehicles. The rail vehicles, which belong to the second group, are called independent traction vehicles. It is so-called a diesel multiple unit (abbr. DMU), it is a multiple-unit train, and it is powered by an on-board diesel engine. An DMU does not need to include a separate locomotive, therefore, an engine is incorporated into one or even in more of wagons. There is usually applied a diesel combustion engine, but, nowadays, some other alternative sources of energy are tested [2-4]. These diesel-powered single unit railway vehicles are further classified depending on their transmission type. In principle, there are recognized following types of DMUs: diesel mechanical multiple unit (abbr. DMMU), diesel-hydraulic multiple unit (abbr. DHMU) and diesel-electric multiple unit (abbr. DEMU).

This work is a continuation of the research project focused on the creation of a calculation model of a railway vehicle transmission system. In this case, the considered railway vehicle is an DHMU, i.e. a railway vehicle with a transmission system including a hydraulic converter. As it will be seen below, the derivation of a mathematical model is similar to the procedure of derivation of a mathematical model of a transmission system with

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a flexible, or friction lamellar clutch. However, the specifics and characteristics of the transmission system with the hydrodynamic converter are described in sections below.

The hydrodynamic torque converter is a type of fluid coupling that transfers rotating power from an internal combustion engine to a rotating driven load. In a rail vehicle with an automatic transmission, the torque converter connects the power source to the load. It is usually located between the engine's flexplate and the transmission [5-9].

The main characteristic of a torque converter is its ability to increase torque when the output rotational speed is so low, it allows the fluid, coming off the curved vanes of the turbine to be deflected off the stator while it is locked against its one-way clutch, thus providing the equivalent of a reduction gear. This is a feature beyond the simple fluid coupling, which can match rotational speed but does not multiply torque and thus reduces power.

A general layout of the components of a DMU is as following (***fig. 1***): an engine (1) as a source of power, a joint shaft, which serves to transmit a rotational movement form the engine crankshaft to the wheels. A fluid coupling (2) is fixed to the output of the engine. The output from the fluid coupling is connected to a cardan shaft fitted with a freewheel (3), which operates during running downhill [10-12]. Further, the freewheel connects to the gearbox (4) and behind it, the power is transmitted to the final drive (5) by a cardan shaft (6).

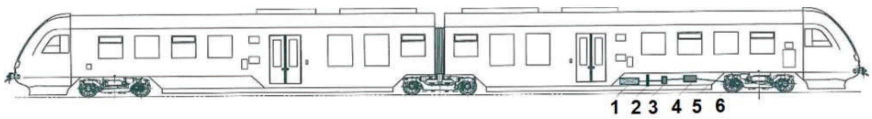

Fig. 1 *An example of a components layout of an DHMU*

Obr. 1 *Príklad rozmiestnenia komponentov DHMU*

2 A SIMPLIFIED MATHEMATICAL MODEL – ASSUMPTIONS AND DERIVATION

The main objective of this work is to derive a simplified mathematical model of an DHMU transmission system. A powertrain of the solved DHMU is a mechanical system with certain typical properties. In our case, the research is focused on investigation of starting the powertrain. Therefore, it is necessary to set-up a simplified mathematical model. It will consist of equations of motion, which are derived by means of a suitable method. The Lagrange’s equations of motion of a second kind method was chosen for this purpose. This method is well known and widely used for derivation of dynamical models of various mechanical systems [13, 14]. It is suitable for mechanical systems performing translational, rotational or combined motions. Its general form is:

$$\frac{d}{dt}\frac{\partial E_K}{\partial \dot{q}_j} - \frac{\partial E_K}{\partial q_j} + \frac{\partial E_D}{\partial \dot{q}_j} + \frac{\partial E_P}{\partial q_j} = Q_j \ , \ j = 1, 2, ..., n \ , \tag{1}$$

where $\boldsymbol{E_K}$, $\boldsymbol{E_D}$ and $\boldsymbol{E_P}$ are kinetic, dissipative and potential energy of the system, respectively, $\boldsymbol{q_j}$ are the generalized coordinates and their time derivations of the system are velocities and accelerations, respectively. Further, $\boldsymbol{Q_j}$ is a vector of external loads and $\boldsymbol{n}$ determines number of degrees of freedom. It is obvious from eq. 1, that number of equations of motion depends on degrees of freedom of a mechanical system. Moreover, the using of this method requires to determine proper values of individual energies.

Therefore, the first step for application of the method is creation of a dynamical model. We come from a scheme depicted in ***fig. 2***. The solved powertrain of the DHMU composes of several components marked by numbers 1 to 6: 1 – an engine, 2 – a hydrodynamic converter, 3 – a gearbox, 4 – a propeller shaft, 5 – a differential, 6 – a distribution gearbox, 7 – drive wheels.

The hydrodynamic converter has two main moving rotating parts – a pump and a turbine. Angular velocity of a pump is ω_P and angular velocity of a turbine is ω_T. Other angular velocities are marked as following: ω_1 – angular velocity of an engine crankshaft, ω_2 – angular velocity of an input shaft to a gearbox, ω_3 – angular velocity of an output shaft of a gearbox (a propeller shaft), ω_4 – angular velocity of a drive axle shaft.

We consider, that all connecting shafts of the mechanical system are rigid. This means, that $\omega_P = \omega_1$ and $\omega_T = \omega_2$ or that $\varphi_P = \varphi_1$ and $\varphi_T = \varphi_2$. Angular velocities of individual rotating components are marked in ***fig. 2***.

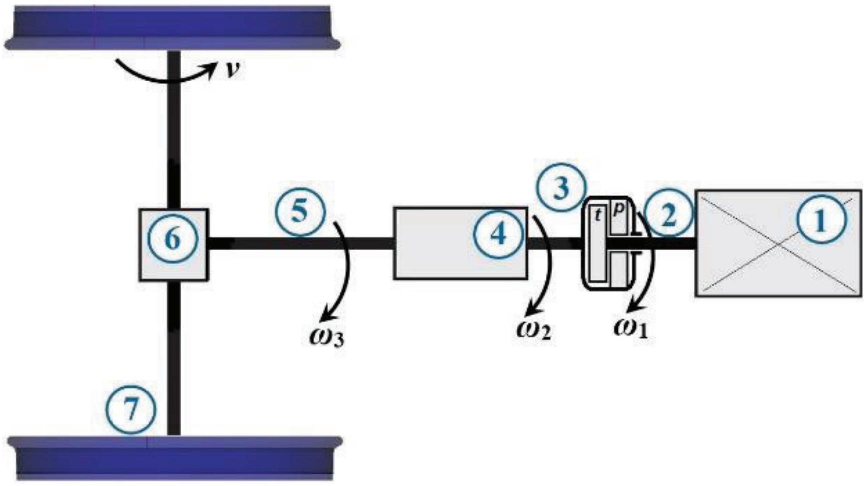

Fig. 2 *A simplified scheme of a mechanical transmission system with an HDC*

Obr. 2 *Zjednodušená schéma hnacej sústavy s HDM*

Thus, we suppose, the mechanical system has two degrees of freedom (2 DOF) and generalized coordinates are φ_1 and φ_2, i.e. angular deviation of the input shaft to the gearbox (the same with the engine crankshaft and the pump shaft) and angular deviation of the output shaft from the gearbox (the same with the turbine shaft).

Kinetic energy of the system is:

$$E_K = \frac{1}{2}\cdot I_e\cdot\omega_1^2 + \frac{1}{2}\cdot I_p\cdot\omega_1^2 + \frac{1}{2}\cdot I_t\cdot\omega_2^2 + \frac{1}{2}\cdot I_4\cdot\omega_3^2 + \frac{1}{2}\cdot I_5\cdot\omega_4^2 + \frac{1}{2}\cdot m_t\cdot v^2 \tag{2}$$

where I_e – moment of inertia of the engine crankshaft [kg·m^2], I_p – moment of inertia of the pump [kg·m^2], I_t – moment of inertia of the turbine [kg·m^2], I_4 – moment of inertia of the shaft between the gearbox and the rear axle differential (a propeller shaft) [kg·m^2], I_5 – moment of inertia of the drive axle shafts [kg·m^2], m_t – the total weight of a rail vehicle [kg] and v – DHMU velocity [m·s^{-2}]. Angular velocities ω_1, ω_2, ω_3, ω_4 are in [rad^{-1}]. As it can be seen, the kinetic energy includes except of generalized coordinates φ_1 and φ_2 also other coordinates. Hence, we consider following relations:

$$\omega_3 = \frac{\omega_2}{i_g}, \quad \omega_4 = \frac{\omega_3}{i_t} = \frac{\omega_3}{i_g\cdot i_d}, \quad v = R\cdot\omega_4 = \frac{R\cdot\omega_2}{i_g\cdot i_d}, \tag{3}$$

where i_t – the total gear ratio, i_g is the gear ratio, i_d is the distribution gearbox ratio and R – the drive wheel radius, which lead to the modified form of the kinetic energy:

$$E_K = \frac{1}{2}\cdot\left(I_e + I_p\right)\cdot\omega_1^2 + \frac{1}{2}\cdot\left[I_t + \frac{I_4}{i_g^2} + \frac{I_5}{\left(i_g\cdot i_d\right)^2} + \frac{m_t\cdot R^2}{\left(i_g\cdot i_d\right)^2}\right]\cdot\omega_2^2 \tag{4}$$

or:

$$E_K = \frac{1}{2} \cdot \left(I_{1red} \cdot \omega_1^2 + I_{2red} \cdot \omega_2^2 \right) \tag{5}$$

where I_{1red} and I_{2red} are moments of inertia of rotational components reduced to the pump shaft and the turbine shaft, respectively.

Dissipative energy expresses viscous losses in the system and it is given as following:

$$E_D = \frac{1}{2} \cdot b_1 \cdot \omega_1^2 + \frac{1}{2} \cdot b_2 \cdot \omega_2^2 \tag{6}$$

where b_1 and b_2 [N·m·s·rad^{-1}] are viscous losses coefficients.

It should be noted, that in the solved task, the DHMU moves on a straight road without a climb. Therefore, potential energy is not being changed, i.e. $E_p = 0$.

Further, in the mechanical system of the powertrain act load moments, namely drive moment of the engine M_e and moment of the driving resistance M_r. Except of them, we consider moments of the pump M_p and the moment of the turbine M_t. These moments have also to be reduced to the corresponding shafts, i.e. to the pump shaft and to the turbine shaft.

The pump torque is given by the formulation:

$$M_p = m_p \cdot K \cdot \left(\frac{n_p}{100} \right)^2 \tag{7}$$

where m_p is the torque coefficient of the pump (given by the hydrodynamic converter producer), K is the hydrodynamic converter factor given by the producer and n_p are revolutions of the pump shaft.

The turbine torque is calculated based on the following equation:

$$M_t = m_p \cdot K \cdot \left(\frac{n_p}{100} \right)^2 \cdot \left(\frac{\eta}{\lambda} \right) \tag{8}$$

where η is the hydrodynamic converter efficiency and λ is revolutions ratio ($\lambda = \omega_t / \omega_p$).

Fig. 3 includes the graph of torques waveforms of the HDC and a scheme of the HDC.

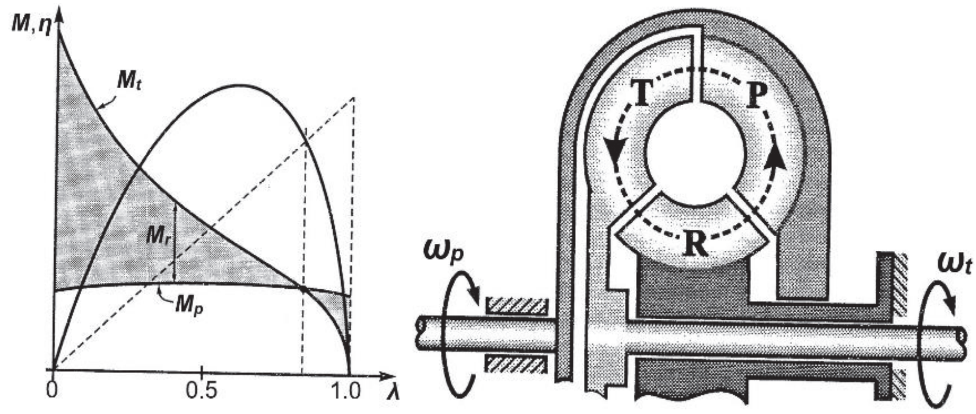

Fig. 3 A *graph of torques waveforms of the HDC (left), a scheme of the HDC (right)*

Obr. 3 *Graf priebehov momentov HDM (left), schéma HDM (vpravo)*

For coordinate $\boldsymbol{\varphi_1}$:

$$M_{1red} \cdot \omega_1 = M_e \cdot \omega_1 - M_p \cdot \omega_p \tag{9}$$

Thus:

$$M_{1red} = M_e - M_p \tag{10}$$

For the coordinate $\boldsymbol{\varphi_2}$ we consider a simplified situation, namely, the DHMU is runs-up from zero speed, i.e. the drag can be neglected and only the weight of the DHMU is considered:

$$M_{2red} \cdot \omega_1 = M_t \cdot \omega_2 - m_t \cdot g \cdot \omega_4 \cdot R = \left(M_t - m_t \cdot g \cdot R \cdot \frac{1}{i_g \cdot i_d} \right) \cdot \omega_2 \tag{11}$$

Thus

$$M_{2red} = M_t - M_r \tag{12}$$

Now, we can perform partial derivations of kinetic and dissipative energies according to the generalized coordinates $\boldsymbol{\varphi_1}$ and $\boldsymbol{\varphi_2}$ at which, the right side of the equations of motion includes reduced moments described above.

Hence, we get the equations of motion of the DHMU transmission system in the following form:

$$\begin{gathered} \left(I_e + I_p \right) \cdot \ddot{\varphi}_1 + b_1 \cdot \dot{\varphi}_1 = M_e - M_p \\ \left[I_t + \frac{I_4}{i_g^2} + \frac{I_5}{\left(i_g \cdot i_d \right)^2} + \frac{m_t \cdot R^2}{\left(i_g \cdot i_d \right)^2} \right] \cdot \ddot{\varphi}_2 + b_2 \cdot \dot{\varphi}_2 = M_t - M_r \end{gathered} \tag{13}$$

or in a shorter form:

$$\begin{gathered} I_{1red} \cdot \ddot{\varphi}_1 + b_1 \cdot \dot{\varphi}_1 = M_{1red} \\ I_{2red} \cdot \ddot{\varphi}_2 + b_2 \cdot \dot{\varphi}_2 = M_{2red} \end{gathered} \tag{14}$$

where $\boldsymbol{I_{1red}}$ and $\boldsymbol{I_{2red}}$ are moments of inertia reduced to the shaft rotating at the angular velocity $\boldsymbol{\omega_1}$ and to the shaft rotating at the angular velocity $\boldsymbol{\omega_2}$, respectively.

They are:

$$\begin{gathered} I_{1red} = I_e + I_p \\ I_{2red} = I_t + \frac{I_4}{i_g^2} + \frac{I_5}{\left(i_g \cdot i_d \right)^2} + \frac{m_t \cdot R^2}{\left(i_g \cdot i_d \right)^2} \end{gathered} \tag{15}$$

The matrix form of equations (11) and (12) is as following:

$$\begin{bmatrix} I_e + I_p & 0 \\ 0 & I_t + \frac{I_4}{i_g^2} + \frac{I_5}{\left(i_g \cdot i_d \right)^2} + \frac{m_t \cdot R^2}{\left(i_g \cdot i_d \right)^2} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} M_e - M_p \\ M_t - M_r \end{bmatrix} \tag{16}$$

or in the shorter form:

$$\begin{bmatrix} I_{1red} & 0 \\ 0 & I_{2red} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \cdot \begin{bmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{bmatrix} = \begin{bmatrix} M_{1red} \\ M_{2red} \end{bmatrix}. \tag{17}$$

3 FINDING AND RESULTS OF CALCULATIONS

Equations of motion derived in the previous section (eq. 13, or eq 14) have been solved in Matlab software. However, these equations of motion should be modified for requirements of this software. Hence, accelerations must be independent. We get:

$$\begin{aligned} \ddot{\varphi}_1 &= \frac{1}{I_{1red}} \cdot \left(M_{1red} - b_1 \cdot \dot{\varphi}_1 \right) \\ \ddot{\varphi}_2 &= \frac{1}{I_{2red}} \cdot \left(M_{2red} - b_2 \cdot \dot{\varphi}_2 \right) \end{aligned} \tag{18}$$

The further modification meets the fact, that the used software is not able to solve differential equations of the second order. Therefore, the equations (16) must be substituted by the differential equations of the first order.

We consider the following substitution for a coordinate φ_1,:

$$\theta_1 = \dot{\varphi}_1, \quad \theta_2 = \varphi_1 \tag{19}$$

Their derivations are as following:

$$\theta_1' = \ddot{\varphi}_1, \quad \theta_2' = \dot{\varphi}_1 = \theta_1 \tag{20}$$

The procedure for the coordinate φ_2 is similar:

$$\theta_3 = \dot{\varphi}_2, \quad \theta_4 = \varphi_2 \tag{21}$$

$$\theta_3' = \ddot{\varphi}_2, \quad \theta_4' = \dot{\varphi}_2 = \theta_3 \tag{22}$$

When the derived equations of motion (13) of the DHMU powertrain are written by means of formulations (19) to (22), we get four differential equations of the first order:

$$\begin{aligned} \theta_1' &= \frac{1}{I_e + I_p} \cdot \left(M_e - M_p - b_1 \cdot \theta_1 \right), \\ \theta_2' &= \theta_1, \\ \theta_3' &= \frac{1}{\left[I_t + \frac{I_4}{i_g^2} + \frac{I_5}{\left(i_g \cdot i_d\right)^2} + \frac{m_t \cdot R^2}{\left(i_g \cdot i_d\right)^2} \right]} \cdot \left(M_t - M_r - b_2 \cdot \theta_3 \right), \\ \theta_4' &= \theta_3. \end{aligned} \tag{23}$$

Parameters of the solved rail vehicle are as following: total weight m_t = 68000 kg, the max. power P = 550 kW, max. torque M_e = 3600 Nm, rolling resistance coefficient f = 0.015.

Fig. 4 depicts the achieved results of the simulation. Outputs values are calculated in the given time interval. Moreover, ***fig. 4*** additionally shows the details of the achieved results in the shorter time interval.

Fig. 4 includes two parts. Torque of the used combustion engine (M_e) and moments of the hydrodynamic converter for the pump part (M_p) and the turbine part (M_t) are in the upper part of the figure.

Operating speed of the input shaft (n_p) and of the output shaft (n_T) together with DHMU speed (v) are shown in the upper part of ***fig. 5***.

After starting the engine and achieving sufficient torque, the torque of the converter pump begins to increase. Together with, the torque of the turbine also begins to increase. In the turbine torque, the moment of loads (i.e. driving resistance) is included. After certain time interval, in this case approx. after 2.8 s, the equilibrium of the torques is reached and then, torques are practically identical.

A similar situation is detected for operating speeds of the pump shaft and the turbine shaft. Operating speeds of the pump shaft $\boldsymbol{n_p}$ begins to increase with the engine torque $\boldsymbol{M_e}$. The operating speed of the turbine shaft $\boldsymbol{n_t}$ are lower than the $\boldsymbol{n_p}$, because in the hydrodynamic converter causes certain decreasing of the operating speed due to losses. The DHMU velocity $\boldsymbol{v}$ is stabilized at the value of 26.64 km/h.

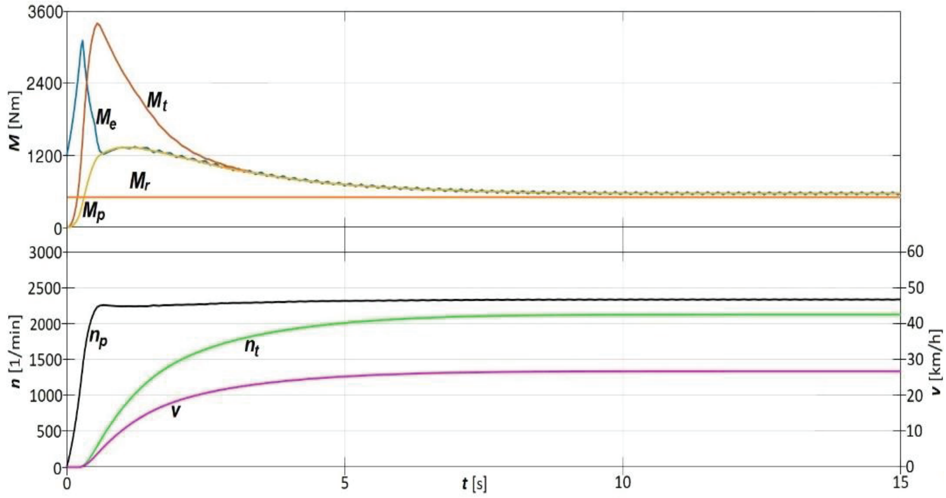

***Fig. 4** The results of calculation of the DHMU transmission system with an HDC*

***Obr. 4** Výsledky výpočtu systému pohonu DHMU s HDM*

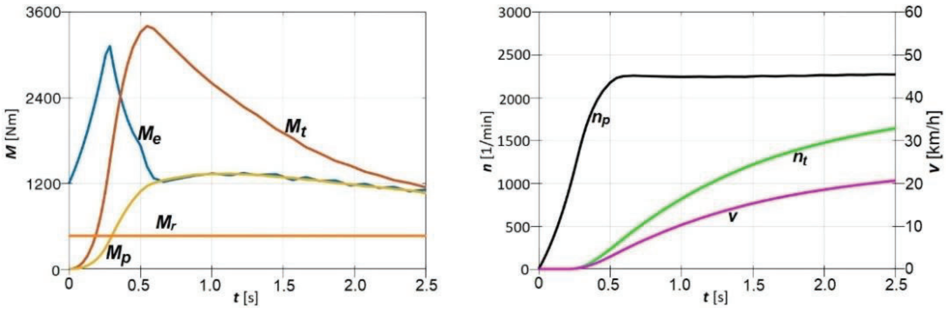

***Fig. 5** The details of the achieved results*

***Obr. 5** Detail dosiahnutých výsledkov*

Waveforms of the outputs are calculated for the one particular activated gear ratio in the gearbox. In the real operation of the DHMU, the gear ratio is changed and the values of torques would decrease and the running speed would be higher. In the case of DHMU running at the higher speed, the drag should be already considered in the mathematical model, and it would be incorporated within the moment of driving resistance $\boldsymbol{M_r}$.

The future research will be focused on modelling the more complex model of an DHMU powertrain. The modified mathematical model will more difficult. It will include other parts for simulation of all running resistances, running in a climb, comparison of outputs values for running with the fully loaded DHMU etc. As the other step of the future research, the setting-up a multibody model (an MBS model) of the solved DHMU. The modelling of the multibody model in the MBS software does not need to derive equations of motion, they are created automatically by a software. On the other hand, it is not possible to check and compare the equations of motion derived by the researcher (let say “manually”) with equations of motion derived by the software during the modelling process.

4 CONCLUSION

The presented research has brought an overview of a creation of a mathematical model for investigation of dynamical properties of a powertrain of an DHMU. The mathematical model was derived by means of the Lagrange’s equations of motion of a second kind method. Calculated values included output torques and operating speed of shafts of the powertrain. Achieved results have shown, that after certain time, the mechanical system gets to the equilibrium. Further change of torques operating speeds of running speed will happen after a change of the gear ratio of a change of the external loads (e.g. a change of running resistance).

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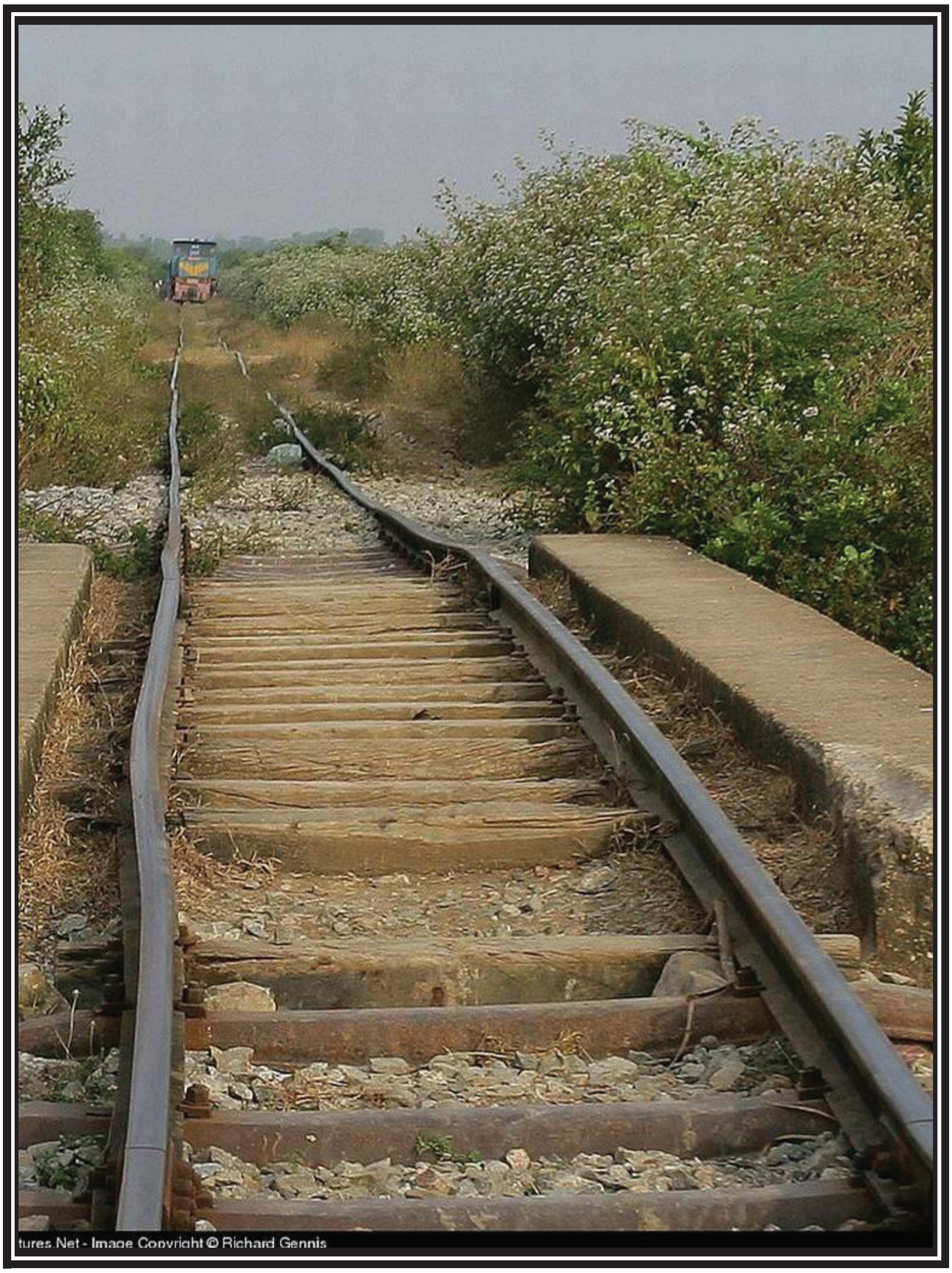
Summary

Mathematical modelling is an undivided part of mechanical engineers' activities. It is used in design, optimization, verifying as well as modifying of products. The approach of creating virtual models and their analysing without having real products is applied in several domains. A mathematical modelling of operation of railway vehicles is one of them. In principle, it is dynamics, which describes mechanical systems in motion. Then, a mathematical model is given by equations of motion. The main objective is to present the way, how a mathematical model of operation of a railway vehicle powertrain with a hydrodynamic converter is derived and used in practical applications. The Lagrange's equations of motion of a second kind method are used for derivation of the mathematical model. A railway vehicle with a hydrodynamic converter was chosen as a referenced vehicle. Derived equations of motion are written in a matrix form and subsequently are solved by means of a technical programming language Matlab. The results of the calculations are presented in a graphical form.

Resumé

Matematické modelovanie je neoddeliteľnou súčasťou činností strojných inžinierov. Používa sa pri navrhovaní, optimalizácii, overovaní ako aj úprave produktov. Prístup tvorby virtuálnych modelov a ich analýzy bez skutočných produktov sa uplatňuje vo viacerých oblastiach. Jednou z nich je aj matematické modelovanie prevádzky koľajových vozidiel. V princípe je to dynamika, ktorá popisuje mechanické systémy v pohybe. Potom je daný matematický model pohybovými rovnicami. Hlavným cieľom je predstaviť spôsob, akým sa odvodzuje matematický model prevádzky hnacej sústavy železničného koľajového vozidla s hydrodynamickým meničom a využíva sa v praktických aplikáciách. Na odvodenie matematického modelu boli použité Lagrangeove rovnice druhého druhu. Ako referenčný vozidlo bolo zvolené železničné vozidlo s hydrodynamickým meničom. Odvodené pohybové rovnice sú zapísané v maticovom tvare a následne sú riešené pomocou technického programovacieho jazyka Matlab. Výsledky výpočtov sú prezentované vo forme grafov.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