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Eigenanalysis of a valvetrain simplified model of a combustion engine with 3DOF

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Abstract: A valvetrain is a necessary structural unit of every combustion engine with discontinued combustion process. It ensures changing cylinder filling, allow low level of exhaust emissions and an optimal combustion process. There are several types of valvetrains depending on their design solution. In principle, there are recognized older valvetrain systems marked as *OHV* (*overhead valve engine*) and *OHC* (*overhead camshaft engine*). Regardless of a type of valvetrain, they include several components, which are in mutual connection. As they are exposed to dynamic effect during operational conditions, they should have suitable dynamical properties. A valvetrain is a mechanical system. The essential dynamical properties of a mechanical system of a valvetrain of a combustion engine include eigenfrequencies and eigenmodes. The article is focused on eigen analysis of a valvetrain of a combustion engine. Equations of motion of a simplified dynamical model of a valvetrain are derived. Eigen characteristics of a chosen type of a valvetrain are calculated. *Matlab* software and *Simpack* multibody software were used for calculation of the derived equations.

Keywords: valvetrain, combustion engine, eigen analysis, equations of motion.

INTRODUCTION

A valvetrain of a combustion engine serves to change cylinder filling. A process of changing cylinder filling, it means, exhaust gas discharge from the combustion chamber and compress space as well as fresh filling supply should be ongoing in exactly defined time intervals regarding to a cylinder position (valvetrain timing) [2, 10].

Valvetrains are mainly installed in four-stroke combustion engines. The most widely used valvetrains in combustion engines are the valvetrains types *OHV*, i.e. overhead valve engine and *OHC*, i.e. overhead camshaft engine. In case of combustion engines with larger dimensions, the *OHV* system is usually applied (Fig. 1). It is because such engines have divided engine head due to design reasons and a camshaft cannot be located [4-6]. The *OHC* valvetrain does not include a push rod, therefore it is stiffer and lighter in comparison with the *OHV* valvetrain. This is its advantage for high-speed engines. From a combustion chamber point of view, both types of valvetrains equivalent, because valves are located in the same way [].

Fig. 1. An example of the *OHV* valvetrain [1]

On the other hand, the *OHC* valvetrain includes a camshaft located in an engine head, which leads to problem to install additional components into it. In principle, two types of *OHC* valvetrain are used in a practice. The *DOHC* (or 2×*OHC*) valvetrain is recognized as a *dual overhead camshaft* (Fig. 2), which includes two camshafts in the engine head. The other type is the *SOHC* valvetrain as a *single overhead camshaft*, which includes only one camshaft in the engine head (Fig. 3) [7-9].

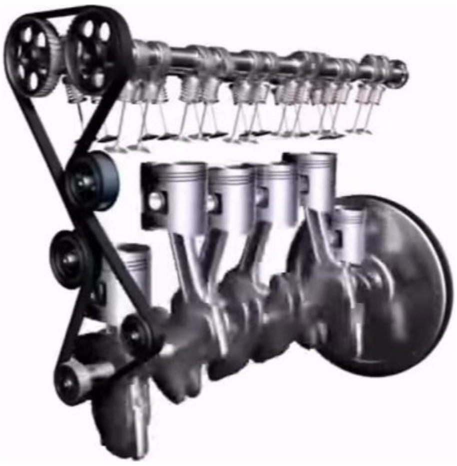


Fig. 2. An example of the *DOHC* valvetrain [1]

Fig. 3. An example of the *SOHC* valvetrain [1]

1 EIGEN ANALYSIS OF A VALVETRAIN

A problem of free oscillation is applied for calculation of eigen frequencies of a valvetrain. They characterized the dynamical properties and oscillation characteristics of a valvetrain in a significant way. Eigen frequencies of a valve train should be as high as possible. This is achieved by a choice of suitable parameters during its design process. The parameters are such as material, stiffnesses, masses and others. Considering that a valvetrain is a complicated oscillating mechanical system, which composes of elements with a character of continuum, it has in the reality infinitely many eigenfrequencies. Therefore, a method for discretization of continuum is applied for eigenanalysis. There is accepted a rule to achieve a sufficient calculation accuracy, that number of discrete masses should be at least twice as high as the number of eigen frequencies. Free oscillation of a valvetrain should be investigated in such a position of a mechanism, in which a valve is open. It means, that clearance between individual components of a valvetrain does not exist. Damping is not considered usually, which partially simplifies a dynamical model of a valvetrain.

The research was solved in two software, namely in *Matlab* and in a multibody software *Simpack*. The results obtained from these software are compared.

Figure 4a depicts a simplified dynamical model of a valvetrain for one cylinder. It includes three masses with elastic couplings between them. Figure 4b shows a multibody model of the considered simplified valvetrain created in the *Simpack* multibody system.

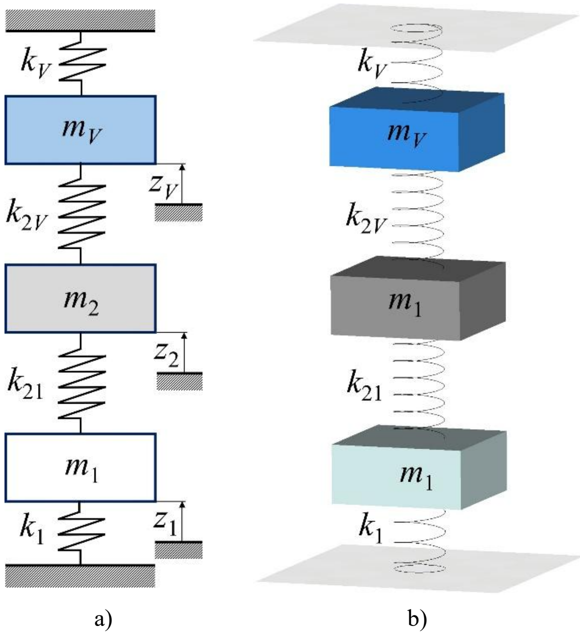


Fig. 4. a) A simplified dynamical model of the solved mechanical system of the valvetrain, b) a multibody model in the *Simpack* software

As it can be seen, the considered mechanical system has three degrees of freedom (3 *DOF*) with considered generalized coordinates x_1, x_2 and x_V. Th equations of motion are derived by means of the method of the *Lagrange's* equations of the second kind, which have the general form for undamped non-forced oscillation as follows [3, 11]:

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}_i}\right) - \frac{\partial E_K}{\partial q_i} + \frac{\partial E_P}{\partial \dot{q}_i} = 0 , \quad (1)$$

where E_K is kinetic energy,
E_P is potential energy,
q_i, $\dot{q}_i$ are generalized velocities and deflections, respectively.

Kinetic energy E_K of the valvetrain simplified mechanical system with 3 *DOF* is:

$$E_K = \frac{1}{2} \cdot \left(m_V \cdot \dot{z}_V^2 + m_2 \cdot \dot{z}_2^2 + m_1 \cdot \dot{z}_1^2 \right), \quad (2)$$

potential energy E_P is:

$$E_P = \frac{1}{2} \cdot \left(k_V \cdot z_V^2 + k_{2V} \cdot (z_1 - z_V)^2 + \right. \\ \left. + k_{21} \cdot (z_1 - z_2)^2 + k_1 \cdot z_1^2 \right), \tag{3}$$

where k_V, k_{2V}, k_{21}, k_1 are stiffnesses of elastic components in the system, respectively.

Kinetic end potential energies are derived based on the equation (1). Then, the resulting equations of motion are as follows:

$$\begin{aligned} m_V \cdot \ddot{z}_V + (k_V + k_{2V}) \cdot z_V - k_{2V} \cdot z_2 = 0 \\ m_2 \cdot \ddot{z}_2 + (k_{2V} + k_{21}) \cdot z_2 - k_{21} \cdot z_1 - k_{2V} \cdot z_V = 0, \\ m_1 \cdot \ddot{z}_1 + (k_{21} + k_1) \cdot z_1 - k_{21} \cdot z_2 = 0 \end{aligned} \tag{4}$$

or in a matrix form:

$$\begin{bmatrix} m_V & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{bmatrix} \cdot \begin{bmatrix} \ddot{z}_V \\ \ddot{z}_2 \\ \ddot{z}_1 \end{bmatrix} + \\ + \begin{bmatrix} k_V + k_{2V} & -k_{2V} & 0 \\ -k_{2V} & k_{2V} + k_{21} & -k_{21} \\ 0 & -k_{21} & k_{21} + k_1 \end{bmatrix} \cdot \begin{bmatrix} z_V \\ z_2 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{5}$$

The solution of the resulting system of equations of motion is considered in a form:

$$\begin{aligned} z_V = z_{V0} \cdot e^{i \cdot \Omega \cdot t} \\ z_2 = z_{20} \cdot e^{i \cdot \Omega \cdot t}, \\ z_1 = z_{10} \cdot e^{i \cdot \Omega \cdot t} \end{aligned} \tag{6}$$

where z_V, z_{20} and z_{10} are amplitudes of eigenvectors of individual masses on the mechanical system of the simplified valvetrain shown in Fig. 4, respectively,

Ω is eigenfrequency of the system,

t is time.

When the supposed solution is substituted to the equations of motion (eq. 4), the following system of algebraic equations is get:

$$\begin{aligned} (k_V + k_{2V} - \Omega^2 \cdot m_V) \cdot z_{V0} - k_{2V} \cdot z_{20} = 0 \\ -k_{21} \cdot z_{10} + (k_{2V} + k_{21} - \Omega^2 \cdot m_2) \cdot z_{20} - k_{2V} \cdot z_{V0} = 0, \\ -k_{21} \cdot z_2 + (k_{21} + k_1 - \Omega^2 \cdot m_1) \cdot z_{10} = 0 \end{aligned} \tag{7}$$

Their matrix form is as follows:

$$\begin{bmatrix} k_V + k_{2V} - \Omega^2 \cdot m_V & -k_{2V} & 0 \\ -k_{2V} & k_{2V} + k_{21} - \Omega^2 \cdot m_2 & -k_{21} \\ 0 & -k_{21} & k_{21} + k_1 - \Omega^2 \cdot m_1 \end{bmatrix} \cdot \\ \cdot \begin{bmatrix} z_{V0} \\ z_{20} \\ z_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{8}$$

This system of algebraic equations has a non-trivial solution, when a determinant of the system equals to zero. It can be written as follows:

$$D = \\ = \begin{vmatrix} k_V + k_{2V} - \Omega^2 \cdot m_V & -k_{2V} & 0 \\ -k_{2V} & k_{2V} + k_{21} - \Omega^2 \cdot m_2 & -k_{21} \\ 0 & -k_{21} & k_{21} + k_1 - \Omega^2 \cdot m_1 \end{vmatrix}, \\ = 0 \tag{9}$$

Lets return to the equations of motion in the matrix form, i.e. to the eq. (5). This equation can be written in a shorter form as follows:

$$\boldsymbol{M} \cdot \ddot{\boldsymbol{z}} + \boldsymbol{K} \cdot \boldsymbol{z} = \boldsymbol{0}, \tag{10}$$

where $\boldsymbol{M}$ is mass matrix,

$\boldsymbol{K}$ is stiffness matrix,

$\boldsymbol{z}$, $\ddot{\boldsymbol{z}}$, $\boldsymbol{0}$ are vectors of generalized coordinates, generalized velocities and vector of the right side, respectively.

When the eq. 10 is multiplied form the left side by the inverse matrix to the mass matrix $\boldsymbol{M}^{-1}$, we get the following equation:

$$\boldsymbol{M}^{-1} \cdot \boldsymbol{M} \cdot \ddot{\boldsymbol{z}} + \boldsymbol{M}^{-1} \cdot \boldsymbol{K} \cdot \boldsymbol{z} = \boldsymbol{0}. \tag{11}$$

Further, we consider a solution of the system of equations of motion (eq. 10) in this form:

$$\boldsymbol{z} = \boldsymbol{z}_i \cdot e^{i \cdot \Omega \cdot t}, \tag{12}$$

where $\boldsymbol{z}_i$ is a vector of amplitudes, i.e. $\boldsymbol{z}_i = [z_V, z_2, z_1]^T$.

After substituting the solution (eq. 12) and its modification, we get the following equation:

$$\boldsymbol{M}^{-1} \cdot \boldsymbol{K} + \Omega^2 \cdot \boldsymbol{E} = \boldsymbol{0}, \tag{13}$$

where Ω^2 is a square of eigenfrequencies,

$\boldsymbol{E}$ is the identity matrix.

Now, we can apply a known mathematical approach to the solution. It means, that calculation of eigenfrequencies and eigenmodes is transmitted to calculation of eigenvalues and eigenvectors of the matrix $\boldsymbol{M}^{-1} \cdot \boldsymbol{K}$.

The following parameters of the valvetrain system are considered: $m_V = 0.47$ kg, $m_2 = 0.0054$ kg, $m_1 = 0.022$ kg, $k_V = 100 \cdot 10^6$ N·m^{-1}, $k_{2V} = 1000 \cdot 10^6$ N·m^{-1}, $k_{21} = 100 \cdot 10^6$ N·m^{-1}, $k_1 = 120 \cdot 10^6$ N·m^{-1}. The task is to calculate eigenfrequencies and eigenmodes of this system. The calculation comes from the derived mathematical model represented by the equation of motion (eq. 4 or eq. 5). The calculation was performed by means of the *Matlab* software. The used calculation script for the task is presented in Appendix.

The resulting eigenfrequencies form the *Matlab* software as well as form the *Simpack* software are listed in Tab. 1. The results for both software are very similar, only minimal deviations are detected. Therefore, it can be stated, that calculations of eigenfrequencies in the *Matlab* software and in the *Simpack* software are the same.

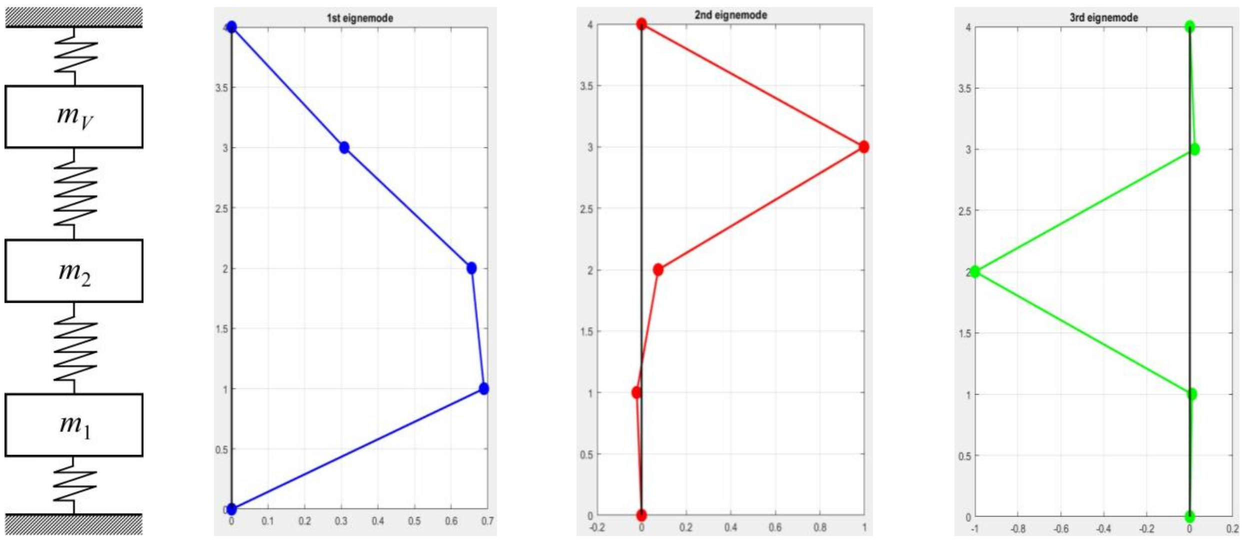
Further, eigenmodes are illustrated in Fig. 5. They are

Tab. 1. The resulting eigenfrequencies achieved from the *Matlab* and *Simpack* software

	Eigenfrequencies *f* [Hz]		Angular eigenfrequencies *Ω* [rad·s^{-1}]	
	Matlab	***Simpack***	***Matlab***	***Simpack***
1st eigenmode	$2.832 \cdot 10^3$	$2.83225 \cdot 10^3$	$1.780 \cdot 10^4$	$17.7956 \cdot 10^3$
2nd eigenmode	$15.643 \cdot 10^3$	$15.6429 \cdot 10^3$	$9.829 \cdot 10^4$	$98.2874 \cdot 10^3$
3rd eigenmode	$72.248 \cdot 10^3$	$72.2478 \cdot 10^3$	$45.395 \cdot 10^4$	$453.946 \cdot 10^3$

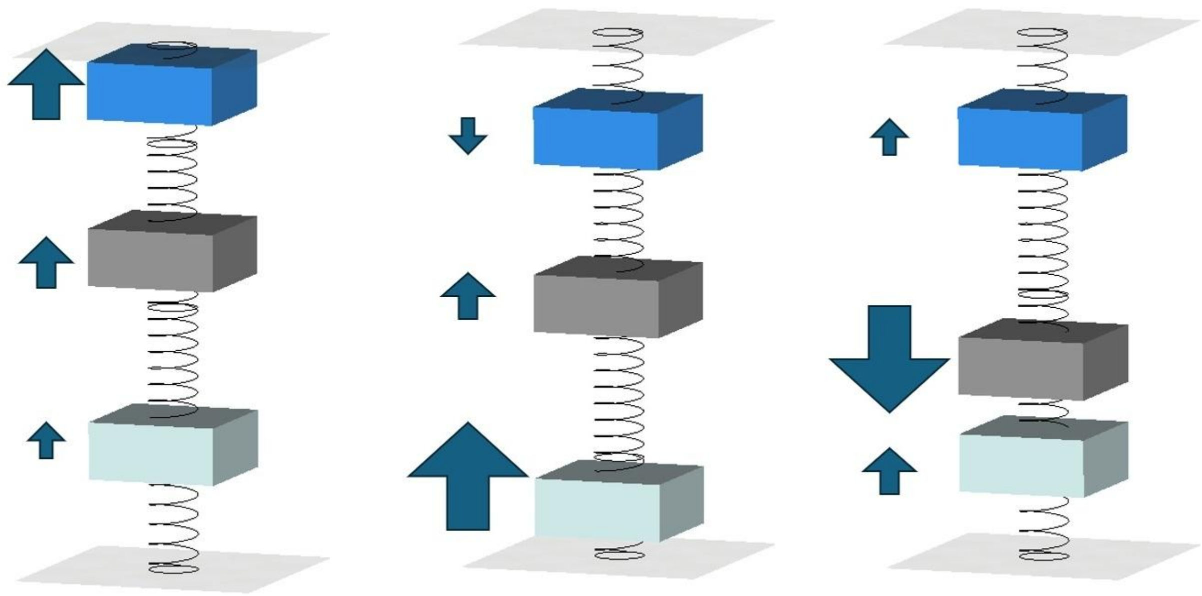
Tab. 2. Calculated values of eigenmodes amplitudes for individual bodies of the valvetrain mechanical system

	m_V	m_2	m_1
1st eigenmode	0.6894	0.6557	0.3078
2nd eigenmode	-0.0217	0.0745	0.9970
3rd eigenmode	0.0104	-0.9997	0.0232



a) b) c) d)

Fig. 5. Illustration of eigenmodes calculated in the *Matlab* software: a) an original mechanical system in an equilibrium position, b) the 1st eigenmode, c) the 2nd eigenmode, d) the 3rd eigenmode



a) b) c)

Fig. 6. Illustration of eigenmodes calculated in the *Simpack* software: a) the 1st eigenmode, b) the 2nd eigenmode, c) the 3rd eigenmode

obtained from the *Matlab* software. As it can be seen, the 1st eigenmodes is without a node, the 2nd eigenmode has one node and the 3rd eigenmode has two nodes. It means, that the mechanical system is in these nudes without movement during oscillation. Further, we can see, that in the 1st eigenmode, the mass m_1 has the biggest deflection, in the 2nd eigenmode, the mass m_V has the biggest deflections and for the 3rd eigenmode, the mass m_2 has the biggest eigenmode. The source commands for creating eigenmodes are written in Appendix.

Figure 6 also shows the eigenmodes of the solved mechanical system of the valvetrain, but they are calculated the *Simpack* software. As it can be seen, they also correspond to the previously calculated eigenmodes.

CONCLUSION

The main goal of the presented research was to calculate the essential dynamical properties of a simplified model of a valvetrain of a combustion engine. Equations of motion were derived by means of the *Lagrange's* equations o the second kind. These equations were written in a ,matrix form and based on the obtained matrices, the eigenmodes and eigenfrequencies were calculated. A multibody model of the solved mechanical system was created in the *Simpack* software. It served for calculation the same dynamical characteristics, i.e. eigenmodes and eigenfrequencies. The resulting numerical values of the eigenfrequencies showed, that both results are very similar. The graphical eigenmodes re also included in the results. The source commands for the *Matlab* software are included in Appendix of the article.

APPENDIX

Source of commands for calculation of *eigenfrequencies* and *eigenmodes* in the *Matlab* software:

```
%% eigenanalysis of valvetrain:
clear all
clc

% input parameters:
mv=0.47;
m2=0.0054;
m1=0.022;
kV=100*10^6;
k2V=1000*10^6;
k21=100*10^6;
k1=120*10^6;

% mass matrix:
M=[mv,0,0;0,m2,0;0,0,m1];

% stiffness matrix:
K=[kV+k2V,-k2V,0;-k2V,k2V+k21,-k21;0,-
k21,k21+k1];

% inverse mass matrix:
MI=inv(M);

% calculation of eigenvalues and
eigenmodes:
[V,D]=eig(MI*K); % V - eigenvectors, D
- eigenvalues (spectral matrix)

Omega_square=sort([D(1,1);D(2,2);D(3,3)
],'ascend');

% angular eigenfrequency:
Omega=sqrt(Omega_square); % [rad/s]

% eigenfrequency:
f=Omega/(2*pi); % [Hz]
```

Source of commands for creating *eigenmodes* in the *Matlab* software:

```
a1=V(:,2); % 1st eigenmode
a2=V(:,3); % 2nd eigenmode
a3=V(:,1); % 3rd eigenmode
y=0:1:4;
r=[0,0];
s=[0,4];
x1=[0,a1',0];
x2=[0,a2',0];
x3=[0,a3',0];

% 1st eignemode - graph
figure(1)
plot(x1,y,'-ob',r,s,'-
k','LineWidth',2,'MarkerEdgeColor','b',
'MarkerFaceColor','b','MarkerSize',10)
title('1st eignemode')
grid

% 2nd eignemode - graph
figure(2)
plot(x2,y,'-or',r,s,'-
k','LineWidth',2,'MarkerEdgeColor','r',
'MarkerFaceColor','r','MarkerSize',10)
title('2nd eignemode')
grid

% 3rd eignemode - graph
figure(3)
plot(x3,y,'-og',r,s,'-
k','LineWidth',2,'MarkerEdgeColor','g',
'MarkerFaceColor','g','MarkerSize',10)
title('3rd eignemode')
grid
```

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