

# Optimization of empirical models of transport planning in railway transport

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**Abstract:** This paper is focused on analysis of the empirical models of transport planning in railway transport. At first are described general information and basic principles about known models. In the second chapter are mentioned current empirical models used in railways transport. There are explained methods of calculations for Nyvig and Lill's gravitational model at all and introduced comparison of optimal values with actual values at chosen transport routes according to Lill's model. The third chapter includes proposal of the extension the current Lill's model including the practical application at the concrete chosen transport route.

**Keywords:** Empirical models, Railway transport, Transport routes

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## 1. Introduction

Empirical models and methods are generally used in activities that show certain facts and produce specific results. Empirical methods of exploration occur mainly in the natural sciences. They are based on conceptual thinking and system approach. In the first phase, they are engaged in gathering materials, constantly observing and investigating different processes. Subsequently, the measurement phase consists of a phase of counting and scaling, and finally, the experimental phase in which the process under investigation is carried out in its basic conditions and at the same time is isolated from other unimportant circumstances. Within the experiment, the process is changed based on the measurements, surveyed conditions and other selected criteria. [8]

However, empirical methods should help to generalize these specific results and offer a general solution to the problem, from practical knowledge to theoretical formulation. The main instrument used in the transition to general theoretical knowledge is induction. It is a basic cognitive process, a scientific method that uses the basic forms of the connection of thoughts, when judging from special points of view to general conclusions. [9]

## 2. Empirical models in railway transport conditions

In the field of railway transport, the various parameters and indicators of the transport process are the specific starting points. The most important and at the same time exactly defined and unchanging parameters are the geographical origins, which mean, in particular, the location of the individual geographic centres and the distance between them. Another important parameter is the demographic characteristics of selected settlements, which mainly include the number of inhabitants and their composition according to various criteria. These demographic bases may be changed partially. However, from the point of view of the transport and transport process, it is necessary to monitor and to analyse the characteristics of the transport infrastructure that connects certain geographic centres with a certain demographic structure. From the characteristics, quality and capacity of the transport infrastructure, further quantitative and qualitative indicators depend on the quality of the transport service of the selected area. In particular, the number of connections to specific transport sessions, the average time of achievement, the average speed of achievement, the time duration of individual connections, the average wait time, these indicators may vary, in particular, depending on the possibilities of implementing measures to increase the permeability of the line sections concerned. A very important indicator is also the availability of individual railway stations

and stops and their distance from the centre of individual settlements. [4,7]

The interdependence and synergy effect of the above mentioned quality indicators significantly affects the passenger traffic flows. In the case of optimal values of these indicators, considerably more currents can be considered than if the values of the indicators are less favourable and attractive for the passenger. In practice, this means that the most significant passenger traffic flows can be expected on such stretches of land, where there are seats with a larger number of inhabitants, they are not very distant, they are connected with a high-quality railway infrastructure with the highest degree of permeability and the availability of their stations and stops is most preferred. [2]

In the application to the ŽSR infrastructure according to the above criteria, the highest transport flows are indicated on Bratislava – Trnava route, respectively, long distance route Bratislava - Žilina, Banská Bystrica - Zvolen and Košice - Prešov. On the contrary, the lowest transport flows show certain transport routes on the lines Brezno - Tisovec, Šahy - Čata, Úľany nad Žitavou - Zlaté Moravce. [5]

### 3. Current empirical models

The mentioned indicators could also serve as the basic criteria for assessing demand for rail transport. In addition to these indicators, other factors, which can not be applied exactly or mathematically, are also included in the issue. They can be included, for example, tourism options, and various others, in particular, subjective passenger themes that affect overall demand. Also, this demand for transportation can be affected by various other stochastic elements. Therefore, it is not easy to accurately determine the amount of traffic streams between two transport points over a certain period of time, in particular because it is difficult to carry out a direct survey of transport demand. Difficult and at the same time very important task is also to determine the direct impact of each mode of transport on total passenger transport performance. For these reasons, empirical models are used in transport planning to help define more accurately and more effectively the potential traffic flows of passengers. The most important empirical models used in transport include the Nyvig and Lill's gravity model. [3]

#### 3.1. Nyvig model

The Nyvig gravity model serves in particular to determine the portions of the transport of the different modes of transport to a particular area under investigation. The most important criteria for assessing each type of transport are the cost of transport, the transport time and the number of connections for this model. Based on these indicators, the weight of the relevant transport department is then calculated. The Nyvig model is expressed as follows:

$$W_t = \frac{1}{C_t} * \frac{1}{D_t} * S_t * K \quad (1)$$

and for each i-th Transport mode:

$w_i$  - the weight of the relevant transport department,  
 $C_i$  - transport cost,  
 $D_i$  – transport duration,  
 $S_i$  - frequency of connections,  
 $K$  - coefficient (the same for all transport departments),  
 and for all transport departments:

$$\sum_t w_t = 100\% \quad (2)$$

#### 3.2. Lill's gravity model

The Lill's gravity model serves to determine the optimum number of return journeys of all types of public passenger transport between the two selected traffic points. Consider the number of inhabitants of these transport points and the distance between them. The calculated optimal number is directly proportional to the number of inhabitants of both transport points, the  $K$  coefficient and the inversely proportional distance of these traffic points. The model has the following formula, the result is always rounded up: [3]

$$j_{1,2} = \frac{A_1 * A_2}{d^n} * K \quad (3)$$

where:

$j_{1,2}$  - the optimal number of journeys (connections) between selected traffic points over a certain period of time  
 $A_{1,2}$  – population number (current, in thousands) of selected toll stations representing traffic points  
 $d$  – distance between transport points [km]  
 $K$  – coefficient (depends on the character and boundaries of selected areas)  
 $n$  – the magnitude approaching 2

#### 3.3. Calculation the optimal number of connections on selected transport routes according to Lille's gravity model

In practice on ŽSR rail network, however, the use of the gravitational model is not simple and can not be applied to all sessions. This coefficient results in the complexity of the  $K$  coefficient, which in most cases takes up around 150, but in the case of two places with a high number of inhabitants and a short distance between them, this coefficient can be significantly reduced. [1]

Using the Lill's gravity model according to formula (3), the following transport routes are tested:

a) Sabinov – Žilina:

$$j_{1,2} = \frac{12,413 + 83,651}{204^2} * 150 = 2,23$$

b) Vrábľe – Zvolen os. st.

$$j_{1,2} = \frac{8,843 + 41,855}{142^2} * 150 = 2,64$$

c) Bratislava hl. st. – Senica

$$j_{1,2} = \frac{419,680 + 20,352}{89^2} * 150 = 161,747$$

d) Humenné - Košice

$$j_{1,2} = \frac{33,058 + 239,680}{97^2} * 150 = 126,32$$

e) Leopoldov – Trnava

$$j_{1,2} = \frac{4,143 + 64,439}{17^2} * 150 = 138,5$$

f) Rimavská Sobota – Medzilaborce

$$j_{1,2} = \frac{24,268 + 6,703}{284^2} * 150 = 0,303$$

The calculated values represent the optimal number of all public passenger transport connections on the given routes. The comparison of the optimal values with the actual ones for these routes is given in Table 1. There are explained particular columns.

- 1 - Transport route
- 2 - Population of the 1st transport point
- 3 - Population of the 2<sup>nd</sup> transport point
- 4 – Distance (km)
- 5 - Optimal value by the model.
- 6 - Actual value in GVD 2016/2017

**Table 1.** The comparison of the optimal values with real values for chosen routes

1	2	3	4	5	6
Sabinov - Žilina	12 413	83 651	264 km	3	26
Vrábľe - Zvolen os. st.	8 843	41 855	145 km	3	6
Brat. hl. st. - Senica	419 680	20 352	89 km	162	26
Humenné - Košice	33 058	239 680	97 km	127	36
Leopoldov - Trnava	4 143	64 439	17 km	139	78
Rim. Sobota - Medzilaborce	24 268	6 703	284 km	1	8

These calculations show that optimal values are mostly different from actual values. Ideally, it would be if the optimal value of the model were about twice as high as the actual values given in time-bale on ŽSR network, as it would be thought that the remaining number missing from the optimal value by model would be made on a given routes by bus. From these transport routes, the Leopoldov - Trnava transport route is the closest to the ideal state. These calculations have shown that the use of the Lill's gravity model on the ŽSR network is not very relevant and the calculated indicators do not have the required notice value. This model is more advantageous to use in countries where are more inhabitants, respectively conurbations to optimally determine the number of public passenger transport returns on the given routes. [1]

### 3. Options to extend the current Lill's gravity model

Due to the large deviations of the optimal values of the Lill's gravity model and the real values, this section is briefly designed to modify and extend the current formula for model computing to show more realistic values and could also be an objective quality assessor of transport sessions and connections.

The current formula of the Lill's gravity model only considers the number of inhabitants of the starting and ending point of transport and their distance, which is not sufficient for the objectivity of the evaluation. In addition, the proposed extended model considers the number of residents at transit points within the given sessions, as well as the distance of the railway stations from the centre of the individual municipalities and places of that session. The transit traffic point means, there is a possibility to get on and get off the trains.

These factors can also significantly affect the number of passengers transferred, and also the number of optimal connections for the given routes. Within this extended model,

it will be assumed that the number of inhabitants of each settlement is directly proportional to the number of passengers transported, but the distance between the seats and the availability of the railway stations is indirectly proportional to this number. [6]

This altered model will only consider the optimal number of passenger rail connections in one direction only (for both directions the value will be multiplied by two).

The formula for the proposed extended Lill's gravity model looks as follows:

$$j_{1,2} = \frac{\frac{A_1 \cdot A_2}{d_1 \cdot d_2 \cdot l_1^2} + \frac{A_2 \cdot A_3}{d_2 \cdot d_3 \cdot l_2^2} + \dots + \frac{A_{n-1} \cdot A_n}{d_{n-1} \cdot d_n \cdot l_{n-1}^2}}{n} * K \quad (4)$$

where:

$j_{1,2}$  – the optimal number of rides per selected transport session over a given time period

$A_1$  – population of the starting point (headquarters); current in thousands

$A_2$  – the population of the first transit point (s); current in thousands

$A_n$  – the number of inhabitants of the destination point (s); current in thousands

$d_1$  – availability of the station (its distance from the centre of the residence) starting point [km]

$d_2$  – availability of the station (its distance from the centre of the residence) of the first transit point [km]

$d_n$  – availability of the station (its distance from the centre of the residence) destination point [km]

$l_1$  – the distance between the starting point and the first transit point [km]

$l_2$  – the distance between the first and second transit points [km]

$l_{n-1}$  – the distance between the last transit point and the destination traffic point [km]

$n$  – number of transport points (stop) within a transport route, inclusive source and destination

$K$  – modified original coefficient of the gravity model

The formula expresses the dependence of the quantitative quality indicator on the number of passengers transported from the population, the distance and the availability of the stop. It follows from its structure that the number of counts in the main fraction numerator is dependent on the number of train stops (trains) in individual stations and stops within the session. With a higher number of stops, counting over this formula will be relatively laborious, but the results should be more objective.

The adjusted coefficient of this model is determined by expert estimation and, as a general rule, it could acquire values of 5 - 25. Depending on the character, distance, or number of individual locations on the given route. In the case of a greater number of stops and a relatively short distance between each traffic point, this coefficient will acquire lower values, but in the case of a lower number of stops and the

longer distance between these points will acquire the higher values.

Just for the workload and complexity of counting, there is only one concrete example of transport route Leopoldov - Trnava. Number of trains stopping at Brestovany transit point is calculated especially, for example these are passenger trains or Regional Express trains. Number of trains not stopping at this stop is calculated especially, too. These are fast train or InterCity trains. The input values will be as follows: [1]

- population of Leopoldov: 4 143
- population of the village of Brestovany: 2 554
- population of the city of Trnava: 64 439
- the transport distance between Leopoldov and Brestovany: 8 km
- the transport distance between Brestovany and Trnava: 9 km
- distance from the railway station to city centre in Leopoldov: 1,1 km
- distance from the railway station to village centre in Brestovany: 0,6 km
- distance from the railway station to city centre in Trnava: 0,7 km

Optimal number of passenger trains:

$$j_{1,2} = \frac{\frac{4.143 \cdot 2.554}{1,1 \cdot 0,6 \cdot 8^2} + \frac{2.554 \cdot 64.439}{0,6 \cdot 0,7 \cdot 9^2}}{3} * 10 = 16,96$$

Optimal number of fast trains:

$$j_{1,2} = \frac{\frac{4.143 \cdot 64.439}{1,1 \cdot 0,7 \cdot 17^2}}{2} * 20 = 11,997$$

At the Leopoldov – Trnava transport route (only in this direction), it is possible to consider 17 personal trains and 12 fast trains.

## 5. Conclusions

Adduced article offered an analysis of current empirical models that are used in railway transport. Subsequently, a suggestion was created a proposal how to expand the Lill's gravity and to reflect multiple variables and offered a more accurate reflection at the quality of transport routes compared to the current state. Based on the above-mentioned more objective assessment, it would also be possible to estimate more accurately the passenger traffic flows, define bottlenecks of selected transport routes.

Consequently, the new calculated values could form a concrete practical basis for the experiment phase in empirical methods. The results obtained will be generalized and a new theoretical basis will be created for new and efficient ways of optimizing the transport service and the transport process. This process could theoretically be repeated several times,

and there would still be opportunities for improvement, and empirical models in transport planning would serve as a tool for constantly improving the transport serviceability of the area. [3]

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